NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 4034

EFFECT OF FLUID-SYSTEM PARAMETERS ON STARTING FLOW IN A LIQUID ROCKET

By Richard P. Krebs

Lewis Flight Propulsion Laboratory
Cleveland, Ohio

Washington
September 1957

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 4034

EFFECT OF FLUID-SYSTEM PARAMETERS ON STARTING FLOW IN A LIQUID ROCKET

By Richard P. Krebs

SUMMARY

An analysis was made to determine the effect of configuration parameters and valve area-time operating relations on the speed of response of the liquid flow and the suppressed suction head of the pump in a rocket motor. The configuration studied consisted of tank, pump, combustion chamber, valves, and lines, including a bypass line with a valve around the pump.

The results show that the area-time relation of the main-flow valve had a most important effect on the suppression head as well as on the speed of response. For a given change in volume flow and given valve operating time, the maximum suppression head was minimized by a flow increase which was linear with time. The maximum suppression head varied directly with the length of the suction line to the pump and inversely with the area of the line and the operating time of the main-flow valve. Friction, changes in tank head, or changes in the length of line from the pump to the rocket chamber had little effect on the speed of response or the suppression head.

INTRODUCTION

In a rocket it is desirable to generate full thrust as soon after firing as possible. A rapid thrust development saves on fuel during the thrust buildup period and eases the guidance problem by providing full thrust before the rocket has departed appreciably from its launching position.

Although a rapid thrust buildup is desirable for the foregoing reasons, the rate of change of thrust is limited by several considerations. Among these are the maintenance of combustion in the rocket chamber and the onset of cavitation in the fuel or oxidant pumps.

The pump cavitation problem is particularly serious. Both quantity flow and delivered head can be seriously reduced by cavitation. Although cavitation can occur any place in the pump where the local pressure falls

below the vapor pressure, the pump inlet is most susceptible. For this reason the pressure drop from the propellant tank to the pump inlet, accompanying a large flow acceleration at starting, and referred to as the suppression head, is of particular interest. This report is concerned chiefly with the development and magnitude of the suppression head.

Factors other than the reduction of the local static pressure below the vapor pressure may have some bearing on the pump performance. For example, the length of time that this local pressure depression prevails may determine how seriously pump performance is affected. If the depression is momentary, the effect on performance may be negligible. On the other hand, even a momentary loss of suction head may so embarrass the pump that it will require considerable time to recover. The deleterious effects of prolonged cavitation are well known. Sudden changes in pump flow may also change the pump characteristics. The magnitude of these transient effects should be determined experimentally and is beyond the scope of this analysis.

The present analysis determines the pertinent pressures and quantity flows as a function of time in a complete rocket configuration. The effect of valve operating-time characteristics, changes in suction-line length, and other parameters was investigated. Although the configuration chosen is not intended as optimum, it serves to illustrate quantitatively some of the difficulties involved in rapid thrust development and the effects of various contributing parameters.

The results were obtained as numerical solutions of a set of simultaneous nonlinear differential equations. The solutions were carried out on a drum-storage digital computer.

CONFIGURATION AND RANGE OF VARIABLES

The configuration to be studied in this report is illustrated in figure 1. It is representative of either the fuel or oxidant system in a liquid bipropellant rocket. This system, while in no way to be considered optimum, has many of the elements to be found in any liquid-rocket-propellant system and is representative of a system used in a missile. Figure 1 shows a tank connected through a line, hereinafter referred to as the "suction line," to the inlet side of a centrifugal pump. The discharge of the pump is connected through a valve to the rocket chamber, and a bypass with a valve is connected around the pump. The flow into the rocket chamber will be called the "main flow"; and the flow through the bypass, the "bypass flow." The drive mechanism for the pump was not considered in this configuration, and the pump was assumed to operate at constant rotational speed.

The bypass system has the advantage of permitting the pump to operate near its design point throughout the valve operating transient. If the bypass flow decreases at about the same rate as the main flow increases, the pump flow remains about constant; and, with constant rotational speed assumed, operation of the pump near its design point is assured. Throughout this report the pump is assumed to be so bypassed that the pump flow is nearly constant.

The assumed pump characteristics, typical of current rocket pumps, are shown in figure 2 for one operating speed. In the numerical analysis a straight line was used to represent the pump characteristics in region 1; an ellipse, in region 2; and a straight line, in region 3. Details of the pump equations are given in appendix B, and symbols are defined in appendix A. For most of the calculations made in this report, the pump operation was limited to regions 1 and 2, where the head varied from 1600 to 1376 feet.

In order to determine the effect of changes in various physical quantities within the system on the response time of the main flow and on the pump suppression head, each of the following quantities was varied individually over the entire range indicated, while the remaining quantities were held at their nominal value:

Quantity	Nominal	Range
Suction-line length, ft	5	2.5 to 10
Suction-line diam., in.	2	1.5 to 3
Main-line friction factor	0.025	0.0125 to 0.05
Valve operating time, sec	0.1	0.1 to 0.2
Pump- to rocket-line length, ft	30	15 to 30
Tank head, ft	96	16 to 176

These nominal values, together with a maximum volume flow of 0.4 cubic foot per second, are representative of either the fuel or oxidant system in a 15,000-pound rocket. During the entire investigation the bypass line was kept constant at a length of 10 feet, a diameter of 2 inches, and a friction factor of 0.025.

Several main-flow valves, each with a different time history of operation, were studied in this analysis. While the nominal valve operating time of 0.1 second is considerably smaller than the operating times being used currently, it probably is compatible with the fastest flow responses which can be tolerated within the limits imposed by structural and combustion considerations. A single bypass valve was used throughout the analysis, because the time history of the bypass flow may have considerable latitude before it has more than a secondary effect on either the main flow or the suppression head.

Prior to the valve operating transient, sufficient flow (about 10 percent of maximum) was entering the thrust chamber to maintain combustion. The fuel-oxidant ratio was such as to establish rated temperature in the chamber, and this ratio and temperature were maintained throughout the valve operating transient. Because the main flow was small at the beginning of the transient, most of the pump flow went through the bypass. At zero time the main valve began to open, and the bypass valve began to close. Values of the main, bypass, and pump flows, the difference in head between the tank and the suction side of the pump (suppression head), and the pressure upstream of the rocket injector were computed as functions of time during the valve operation and until steady-state operating conditions had been established.

ANALYSIS AND RESULTS

The equations describing the flow in the rocket system were developed by equating the pressure drop in a line to the force required to accelerate a mass of fluid plus the force dissipated in the line friction. The compressibility of the fluid, the resilience of the line, and the dead time in the combustion chamber were not considered. The detailed assumptions and complete derivations are given in appendix C.

Effect of Valve Resistance-Time History on Pump Suppression Head and Flow Response Time

In order to analyze the effect that the time variation of the main-valve resistance has on the pump suppression head and the flow response time, reference is made to two equations for the system which describe the time derivative of the main flow and the pump suppression head. These equations are taken from appendix C:

$$\frac{L_s + L_d + L_e}{gA_a}\dot{Q}_a = \Delta h_p + h_1 - \left[\frac{f_a(L_s + L_d + L_e)}{2gD_aA_a^2} + R_c + R_a\right] Q_a^2 - GQ_a \tag{1}$$

$$h_1 - h_3 = \frac{L_s}{gA_a}\dot{Q}_a + \frac{f_aL_s}{2gD_aA_a^2}Q_a^2 \tag{2}$$

where R_c and R_a are the resistance of the injector and the resistance of the main valve, respectively. (Symbols are defined in appendix A.) The resistance of the main valve, of course, varies with time. Resistance is used in this report as the ratio of the change in head across the element to the square of the volume flow through it.

It is apparent from equation (1) that the time history of the main-flow valve resistance R_a may be an important factor determining the magnitude of the rate of change of the main flow. Accordingly, the effect of several theoretical valves on the rate of change of flow and the suppression head was studied. All these valves were fast-acting with an assumed operating time, in most cases, of 0.1 second; the time rate of change of area was always zero or positive.

Minimum obtainable suppression head. - For the values of the parameters used in most of this analysis ($f = 0.025$, $L_a = 5$ ft, $D_a = 2$ in., and maximum flow = 0.4 cu ft/sec), the second term on the right side of equation (2) is small compared with the term involving $\dot{Q}_a$. Accordingly, the suppression head is almost proportional to the rate of change of flow, and the suppression head will be a minimum when $\dot{Q}_a$ is a minimum.

The change in the main flow, ΔQ_a in figure 3, can be expressed as the time integral of the rate of change of flow with the limits between 0 and Δt_v:

$$\Delta Q_a = \int_0^{\Delta t_v} \dot{Q}_a \, dt$$

In order to satisfy the foregoing equation and at the same time have as small a maximum value as possible in the valve operating interval,

$$\dot{Q}_a = \frac{\Delta Q_a}{\Delta t_v} = \text{Constant} \tag{3}$$

Because of the inertia of the fluid, the value of Q_a at the end of the valve operating time will be somewhat less than the final value. In order to find $\dot{Q}_a$ and ΔQ_a, equations (1) and (3) must be solved simultaneously for Q_a and $\dot{Q}_a$ at the end of the valve operating time when R_a has reached its final value.

As an example, consider a typical configuration where

$L_s = 5$ ft	$Q_a = 0.04$ cu ft/sec at $t = 0$
$D_a = 0.1667$ ft	$L_d + L_e = 30$ ft
$f_a = 0.025$	$A_a = 0.0218$ sq ft
$h_l = 96$ ft	$\Delta h_p = 1600$ ft

At design conditions,

$$Q_a = 0.4 \text{ cu ft/sec}$$

$$h_8 = 1152 \text{ ft}$$

$$\Delta h_c = 288 \text{ ft}$$

Referring to equation (1),

$$\frac{L_s + L_d + L_e}{gA_a} = 49.8$$

$$R_c = \frac{\Delta h_c}{Q_a^2} = \frac{288}{0.16} = 1800$$

and

$$\frac{f_a(L_s + L_d + L_e)}{2gD_aA_a^2} + R_c = 1971$$

Also,

$$G \equiv \frac{h_8}{Q_a} = \frac{1152}{0.4} = 2880$$

With the preceding results, equation (1) becomes

$$49.8\ \dot{Q}_a = 1696 - (1971 + R_a)Q_a^2 - 2880Q_a \tag{4}$$

The initial and final values of the valve resistance R_a can be computed from equation (4) by setting $\dot{Q}_a = 0$ and $Q_a = 0.04$ and 0.4, respectively.

$$(R_a)_{initial} = 985{,}000$$

$$(R_a)_{final} = 1428$$

As mentioned previously, equation (4) with $R_a = 1428$ and equation (3) with $\Delta t_v = 0.1$ and $(Q_a)_{initial} = 0.04$ can be solved simultaneously for $\dot{Q}_a$ and Q_a at the time the valve is fully open. The following values were obtained:

$$Q_a = 0.37 \text{ cu ft/sec}$$

at the end of the valve operation, and

$$\dot{Q}_a = 3.3 \text{ cu ft/sec}^2$$

Substitution of these values in equation (2) with

$$\frac{L_s}{gA_a} = 7.12$$

and

$$\frac{f_a L_s}{2gD_a A_a^2} = 24.5$$

gives a suppression head of about 27 feet at the end of the valve operating time. Because the rate of change of flow $\dot{Q}_a$ is constant, and the first term on the right side of equation (2) is the controlling term, the suppression head is approximately equal to 27 feet during the entire valve operating time if the main flow is as shown in figure 3. This is the smallest value of the maximum suppression head which can be expected with this configuration and any valve which has an operating time of 0.1 second. Because the flow shown in figure 3 gives the smallest value of maximum suppression head, it will hereinafter be referred to as the "ideal flow."

The ideal flow just described is not physically realizable. The discontinuity in $\dot{Q}_a$ at $t = 0$ requires a discontinuity in R_a at the same time. Reference to equation (1) or (4) indicates that for $t \leq 0$ the required value for R_a is determined with $\dot{Q}_a = 0$. For a time just greater than zero, $\dot{Q}_a$ has some value other than zero (3.3 in the example given), and the equations require a value of R_a less than that computed for $t < 0$. In the example cited, this step change in R_a at $t = 0$ is only about 10 percent of the initial value. However, if the head loss due to the acceleration of the fluid were larger with respect to the over heads involved in equation (1) or (4), then a larger step change in R_a would be required to realize the ideal flow and minimum suppression head.

Maximum obtainable suppression head. - The largest suppression head which could be encountered with this configuration is obtained by using a main valve whose resistance jumps instantaneously from the initial to the final value. At an initial value of $Q_a = 0.04$ cubic foot per second, the valve resistance R_a is changed from 985,000 to a final value of 1428. By substituting the initial Q_a and the final R_a in equation (1), the maximum value of $\dot{Q}_a$ is found to be 31.6 cubic feet per second squared. Substitution of this value of $\dot{Q}_a$ in equation (2) gives a

maximum suppression head of about 225 feet. The relation between the main flow and time when a step valve is used is shown in figure 4.

Flow response time. - The flow response time, similar to the response time used to describe the time-voltage change on a condenser being charged through a resistor from a constant source, is, by definition, the time required for 63 percent of the total flow change to occur. For the step valve (fig. 4), for example, the flow response time is a little more than 0.01 second.

Two valves that have step functions for their area change could completely encompass the valve resistance-time domain for all valves having operating times of 0.1 second. The first of these valves immediately changes from its initial to final value of resistance. Such a valve will give quick response time (0.01 sec, fig. 5) but a large suppression head. The other valve waits until the very end of the valve operating time and then opens instantaneously. This valve gives a very slow response time (0.11 sec) and has the same large suppression head. The linear flow (constant rate of change of Q_a during the valve operation) is also shown in figure 5. It gives a response time of 0.069 second and a suppression head of 27 feet.

Thus, flow response times between 0.01 and 0.11 second can theoretically be obtained with valves having operating times of 0.1 second. In order to obtain the smallest value of maximum suppression with each speed of flow response, certain restrictions are put on the flow. For example, if the flow response time is less than that of the ideal flow, as indicated by point B in figure 6, the smallest value of maximum suppression head will occur when the flow increases linearly with time from point A to point B. If the response time for the flow exceeds that for the ideal, such as at C, then the flow must increase linearly from point C to the time of the end of valve operation, point D, in order to obtain the smallest value of maximum suppression head with this flow response time. The variation of the flow with time from B to E and from A to C has no effect on the maximum value of the suppression head so long as the rate of change of flow from B to E does not exceed that from A to B; or so long as the rate of change of flow from A to C does not exceed that from C to D. These conclusions are valid as long as the head loss due to friction in the suction line is small compared with that due to the acceleration of the flow.

Calculations were made to determine the smallest maximum suppression heads obtainable for each of a range of flow response times. The results are plotted as the solid curve in figure 7. This curve indicates the minimum penalty in suppression head resulting from a valve which opens slower or faster than that valve which gives an ideal flow. Actually, the curve is for the optimum valve movement for each response time. Any other valve time history will result in a point on figure 7 somewhere

above the solid curve. The dotted line at a maximum suppression head of 225 feet indicates the upper boundary for the suppression head. All valves would yield a combination of response time and maximum suppression head within the area bounded by the dotted line and the solid curve.

In connection with figure 7, a number of flow solutions were obtained with widely varying valve resistance-time histories (fig. 8). These main-valve resistances were generated from simple mathematical functions which are listed in table I. The resulting values of maximum suppression head and flow response time are plotted in figure 7. The numerals beside each data point correspond to the resistances shown on figure 8, and the flow and area relations shown on figure 9. Although a wide variety of valve resistances is represented, the points generally follow the optimum curve.

For all the valves in the present analysis, the theoretical valve flow area is quite small (less than 0.03 sq ft at the full-open position) compared with the area of the pipe in which it is located. For such a valve the head loss across the valve is given to a first approximation by the formula for loss due to sudden expansion (ref. 1):

$$\Delta h \approx \left(\frac{1}{A_v} - \frac{1}{A_a}\right)^2 \frac{Q_a^2}{2g} \tag{5}$$

By definition,

$$R_a = \frac{\Delta h}{Q_a^2} \tag{6}$$

Substituting equation (6) in equation (5) gives

$$R_a \approx \frac{1}{2g}\left(\frac{1}{A_v} - \frac{1}{A_a}\right)^2$$

or

$$A_v \approx \frac{1}{\dfrac{1}{A_a} + \sqrt{2gR_a}} \tag{7}$$

Equation (7) gives the approximate valve area as a function of the pipe area and the valve resistance. Figure 9 shows the approximate main-valve-area variation and the main-flow variation expressed as ratios of the maximum area and flow, respectively, for each of the valve resistance variations of figure 8.

The bypass-valve resistance variation (fig. 10) was not altered in the preceding series of solutions. The chief effect of the bypass valve

is to control the volume flow through the pump. If the bypass valve closes approximately as the main valve opens, the flow through the pump will not fluctuate appreciably. Unless the fluctuation is sufficient to change Δh_p, there is no effect on the suppression head.

Details of suppression head and system flows. - Two main-flow valves with operating times of 0.1 second that gave low values of maximum suppression head are valves 6 and 1 (fig. 9). The main flow, bypass flow, pump flow, and suppression head that result from use of main-flow valve 6 and the bypass valve are shown in figure 11 as functions of time. The flow required for minimum suppression head has been copied from figure 5 and is shown as a dashed curve. The main flow with the valve under consideration approximates this ideal flow. It is slower in starting and therefore has to accelerate through the midportion of the valve operating time. 4516

The suppression head accurately reflects the slope of the main-flow curve. The suppression head is initially low where the flow curve is relatively flat. At about 0.025 second, the slope of the flow curve is equal to that of the ideal-flow curve, and the suppression head is about 27 feet. The slope of the flow curve increases, reaching its maximum value at about 0.05 second. The suppression head reaches its maximum value of about 38.4 feet at the same time. As the valve continues to open, the slope of the flow curve decreases and so does the suppression head. The final value of the suppression head is that due to the line friction.

Another main-flow valve resistance which gave a low suppression head is that plotted in figure 9(a) (valve 1). Initially, the resistance of valve 1 decreases much more rapidly than the resistance of valve 6 (fig. 8), and the flow responds more quickly (figs. 9(a) and (f), respectively). As a result, the suppression head builds up rapidly to a value of about 7 feet in 0.001 second (fig. 12). This is a favorable tendency since, as has been shown previously, the suppression head for the linear flow would immediately take on a value of 27 feet and remain at that value during the valve operation. The maximum suppression head with valve 1 was 34.5 feet, the lowest value obtained with any valve investigated.

Effect of total valve operating time. - An additional calculation was made in which the total valve operating time for both the main valve and the bypass valve was increased to 0.15 second. The variation of main-valve resistance with time was similar to that shown for valve 1 in figure 8 except that the time was multiplied by 1.5; the bypass resistance was similar to that shown in figure 10 with the time scale multiplied by 1.5. In an analogous manner, a calculation was made for a total valve operating time of 0.2 second.

The main flow for each of the calculations is shown in figure 13. The corresponding suppression heads are shown in figure 14. As the valve operating time is extended from 0.10 to 0.20 second, the rate of change of flow is reduced. The response time is increased from about 0.074 second with the 0.10-second valve to 0.143 second with the 0.20-second valve. The corresponding maximum suppression heads are 34 and 19 feet. If the maximum suppression head is plotted against the inverse response time, as has been done in figure 15, the result is a straight line.

Effect of Configuration Parameters

The effect of various parameters other than R_a on the flow and the suppression head was investigated. The main-valve configuration of figure 9(a) was used throughout the following portions of the analysis.

Length of suction line. - The effect of changes in the length of the suction line (line s, fig. 1) was investigated. Suction-line lengths of 2.5, 5.0, and 10.0 feet were used. The effect of these changes (l_s = 2.5 and 10.0) on the main flow Q_a was very slight (fig. 16(a)).

The suppression head for each suction-line length is shown in figure 16(b) as a function of time. The general shape of the three curves in figure 16(b) is similar. Increasing the length of the suction line magnifies the effect of changes in the flow derivative.

The maximum suppression head is shown as a function of suction-line length in figure 17. Equation (2) indicates that the maximum suppression head is directly proportional to suction-line length if the maximum values of Q_a and $\dot{Q}_a$ remain the same.

Effect of tank head. - It should be pointed out that such suppression heads as are indicated in figures 16(b) and 17 will be realized only if the local pressure at the pump inlet has remained above the vapor pressure of the liquid being pumped. If this condition is not met, then cavitation will take place in the pump. Cavitation changes the pump characteristics, such as flow capacity and head. There is no provision in the analysis for a cavitating pump, and the results are invalid once cavitation sets in. In case the suppression head is greater than the tank head, cavitation obviously will render the present results invalid unless the tank head is increased. Changing the tank head, however, has little effect on either the flow or the suppression head. An increase in tank head from 96 to 176 feet changes these two quantities less than 3.5 percent. The small effect of changes in tank head on the flow and maximum suppression head is attributed to the fact that the pump head is large compared with the tank head.

Area of suction line. - The same configuration with a 5-foot suction line was used to investigate the effect of suction-line area on the maximum suppression head. Suction lines with diameters of 1.5, 2, and 3 inches were used. The maximum suppression head is plotted against the inverse suction-line area in figure 18. The relation between the two is very nearly linear. Here again, this result follows only in systems in which the total resistance to the main flow comes chiefly from resistances downstream of the pump.

Friction factor. - The same configuration was used to study the effect of friction factor. The friction factor in the suction line was doubled and halved from its nominal value of 0.025. Changes in the suppression head were less than 0.2 of 1 percent. These results indicate that, for the configuration considered, friction in the suction line was of negligible importance.

Length of line from pump to rocket chamber. - In some rockets the combustion chamber is cooled by passing liquid propellant through a coil wrapped around the chamber. The length of this coil is determined, in part, by the amount of cooling required. Since this coil is in series with the line from the pump to the rocket chamber, it may have some effect on the flow response time and on the maximum suppression head. The magnitude of this effect was studied by reducing the pump-to-chamber line from a nominal value of 30 feet to 15 feet. No other part of the configuration was changed. The main flow and the suppression head for these two pump-to-chamber-line lengths are shown as functions of time in figure 19.

Figure 19 shows that shortening the line decreased the response time by less than 0.002 second. Accompanying this small decrease in response time was an increase in maximum suppression head of approximately 2.5 feet.

CONCLUSIONS

An analysis has been made of a possible rocket fuel or oxidant system during the starting transient while the valves are changing area. From this analysis, the following conclusions may be drawn regarding this system:

1. The time history of the valve resistances is important in determining not only the speed of response of the flow, but also the maximum suppression head on the suction side of the pump.

2. A domain exists in the response-time - maximum-suppression-head plane which encompasses the behavior of all main-flow valves with a fixed operating time and a zero or positive rate of change of area. Several valves were investigated with performances which fell near the favorable borders of this domain.

3. For a given flow change and valve operating time, the maximum suppression head is directly proportional to the length of the suction line to the pump and nearly inversely proportional to the area of the suction line and the total valve operating time.

4. Friction in the line, changes in the tank head, and changes in the length of line between the pump and the rocket chamber had little effect on the flow or the suppression head as long as cavitation was avoided.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, June 11, 1957

APPENDIX A

SYMBOLS

A	line area, sq ft
A_v	valve area, sq ft
C_1, C_2	parameters used to describe valve resistance
D	line diameter, ft
f	friction factor
G	constant
g	acceleration due to gravity, 32.2 ft/sec^2
h	head, ft
Δh	change in head, ft
L	line length, ft
p	pressure, lb/sq ft
Q	quantity flow, cu ft/sec
ΔQ_a	change in Q_a during total valve operating time
R	resistance, ratio of change in head across element to square of volume flow through it, sec^2/ft^5
t	time, sec
Δt_v	total valve operating time, sec
v	velocity, ft/sec
w	weight flow, lb/sec
ρ	density, lb/cu ft

Subscripts:

a	main line or valve

4516

b	bypass line or valve
c	injector
d	from pump discharge to main valve
e	from main valve to injector
max	maximum
p	pump
s	suction line
1	tank
3	pump inlet
4	pump outlet
5	main-valve inlet
6	main-valve outlet
7	injector, high pressure
8	rocket chamber

APPENDIX B

PUMP EQUATIONS

In a typical liquid pump, the maximum head $h_{p,max}$ produced at some quantity flow less than maximum is proportional to the square of the pump rotational speed αN^2. The maximum flow Q_{max} delivered at zero head is proportional to the first power of the speed mN. At any given rotational speed, the pump head varies with the quantity flow as shown in figure 2.

For this analysis the dependence of pump head on flow has been divided into three regions, and each region has been described by a different algebraic relation. In region 1 the head is independent of the flow and equal to the nominal, or maximum, head developed by the pump at a given rotational speed:

$$h_p = \alpha N^2$$

In the second region the dependence was approximated by an ellipse with the major axis a portion of the maximum flow and the minor axis a portion of the nominal pump head:

Major axis:

$$(1 - \beta - \varepsilon)Q_{max} = (1 - \beta - \varepsilon)mN$$

Minor axis:

$$\gamma h_{p,max} = \alpha\gamma N^2$$

The explicit expression for the pump head, in region 2, is

$$h_p = \left[(1 - \gamma) + \gamma \sqrt{1 - \frac{(Q_p - \beta mN)^2}{(1 - \beta - \varepsilon)^2 m^2 N^2}}\right]\alpha N^2$$

When the flow exceeds $(1 - \varepsilon)Q_{max}$, the pump head falls off precipitously from a value of $\gamma\alpha N^2$ to zero over a change in flow equal to εQ_{max}. A linear fall in head has been assumed over this small flow range, so that, in region 3,

$$h_p = \frac{mN - Q_p}{m\varepsilon}\alpha N(1 - \gamma)$$

For any flow greater than Q_{max}, the pump head was assumed equal to zero.

The following values of parameters were used in this report:

$$\alpha = 2.56\times10^{-6} \qquad N = 25{,}000$$

$$\beta = 0.8 \qquad \varepsilon = 0.005$$

$$\gamma = 0.14 \qquad m = 25\times10^{-6}$$

4516

CG-3

APPENDIX C

DIFFERENTIAL EQUATIONS OF MOTION

With an incompressible flow assumed through a pipe of constant area, the pressure drop Δp required to accelerate the fluid against the friction force in a pipe of length L is

$$\Delta p = \frac{\rho L}{g} \frac{dv}{dt} + \frac{f \rho v^2 L}{2gD}$$

or

$$\Delta h = \frac{L}{g} \frac{dv}{dt} + \frac{f v^2 L}{2gD} \tag{C1}$$

where

$$\Delta h = \frac{1}{\rho} \Delta p$$

In terms of volume flow $Q = vA$, equation (C1) becomes

$$\Delta h = \frac{L}{gA} \dot{Q} + \frac{fL}{2gD} \left(\frac{Q}{A}\right)^2 \tag{C2}$$

where $\dot{Q}$ denotes dQ/dt.

The loss in head across the injector and the valves was assumed proportional to the square of the volume flow:

$$\Delta h = RQ^2 \tag{C3}$$

where R is the resistance of the element.

The temperature in the rocket chamber was assumed constant, and the throat was assumed choked. For choked flow at constant temperature, the weight flow is proportional to the chamber pressure:

$$w = \frac{p_8}{G} = \frac{\rho h_8}{G}$$

But $w = \rho vA = \rho Q$; therefore,

$$Q = \frac{h_8}{G} \tag{C4}$$

so that h_8 is proportional to Q.

4516

With these assumptions, and by reference to figure 1, the following equations were written for the configuration: Between the tank and the suction side of the pump, the drop in head $h_1 - h_3$ is the result of the acceleration of the flow and the friction in the line to the flow:

$$h_1 - h_3 = \frac{L_s}{gA_a}\dot{Q}_a + \frac{f_a L_s}{2gD_a A_a^2} Q_a^2 \tag{2}$$

In the bypass circuit the head across the pump is equal to the loss in head due to flow acceleration, friction, and the resistance of the bypass valve:

$$\Delta h_p = h_4 - h_3 = \left(\dot{Q}_b + \frac{f_b Q_b^2}{2D_b A_b}\right)\frac{L_b}{gA_b} + R_b Q_b^2 \tag{C5}$$

The head at the discharge side of the pump is equal to the sum of the head in the rocket chamber, the head across the main valve and the injector, and the head lost in accelerating the flow and in friction in the line from the pump to the rocket chamber:

$$h_4 = GQ_a + (R_a + R_c)Q_a^2 + \left(\dot{Q}_a + \frac{f_a Q_a^2}{2D_a A_a}\right)\frac{L_d + L_e}{gA_a} \tag{C6}$$

The last relation required equates the pump volume flow to the sum of the main flow and the bypass flow:

$$Q_p = Q_a + Q_b \tag{C7}$$

where R_a and R_b are functions of time, and h_p is a function of Q_p as given in appendix B. From these equations it follows that

$$\frac{L_s + L_d + L_e}{gA_a}\dot{Q}_a = \Delta h_p + h_1 - \left[\frac{f_a(L_s + L_d + L_e)}{2gD_a A_a^2} + R_c + R_a\right] Q_a^2 - GQ_a \tag{1}$$

$$\frac{L_b}{gA_b}\dot{Q}_b = \Delta h_p - \left(\frac{f_b L_b}{2gD_b A_b^2}\right)Q_b^2 - R_b Q_b^2 \tag{C8}$$

$$h_7 = R_c Q_a^2 + GQ_a \tag{C9}$$

Equations (C7), (1), and (C8) were solved simultaneously with the aid of the relations in appendix B. Equations (2) and (C9) were solved for the suppression head on the suction side of the pump and for the pressure in the rocket chamber, respectively.

REFERENCE

1. Hunsaker, J. C., and Rightmire, B. G.: Engineering Applications of Fluid Mechanics. McGraw-Hill Book Co., Inc., 1947.

4516

TABLE I. - MATHEMATICAL EXPRESSIONS

DESCRIBING MAIN-VALVE RESISTANCE

CHANGES WITH TIME

Valve	Resistance
1	$C_2 e^{C_1 t}$
2	$\frac{1}{C_1 + C_2 t^2}$
3	$C_1 + C_2 t$
4	$\frac{1}{C_1 + C_2 t^8}$
5	$\frac{1}{C_1 + C_2 t^{16}}$
6	$\frac{1}{C_1 + C_2 t^4}$
[a]7	$\frac{1}{C_1 + C_2 t}$

[a]The variation of the bypass resistance R_b with time had the same form as that for valve 7.

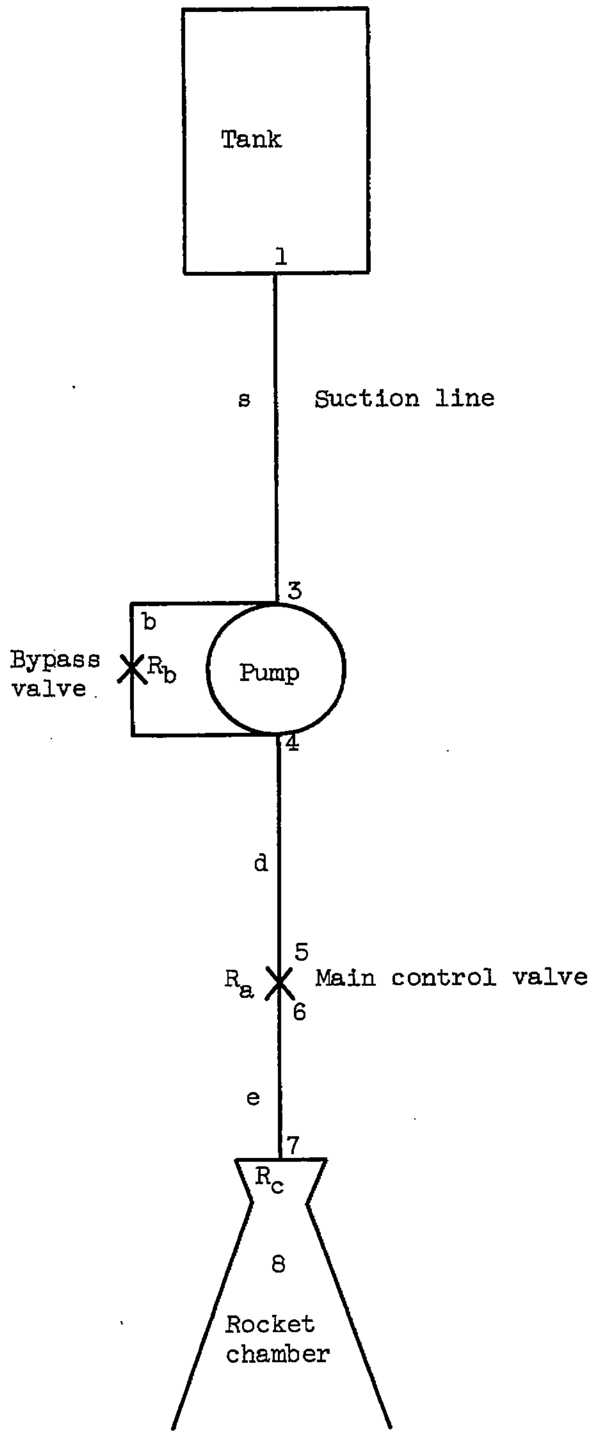


Figure 1. - Schematic diagram of rocket configuration. (Letters refer to lines and numerals to junctions of lines and system elements.)

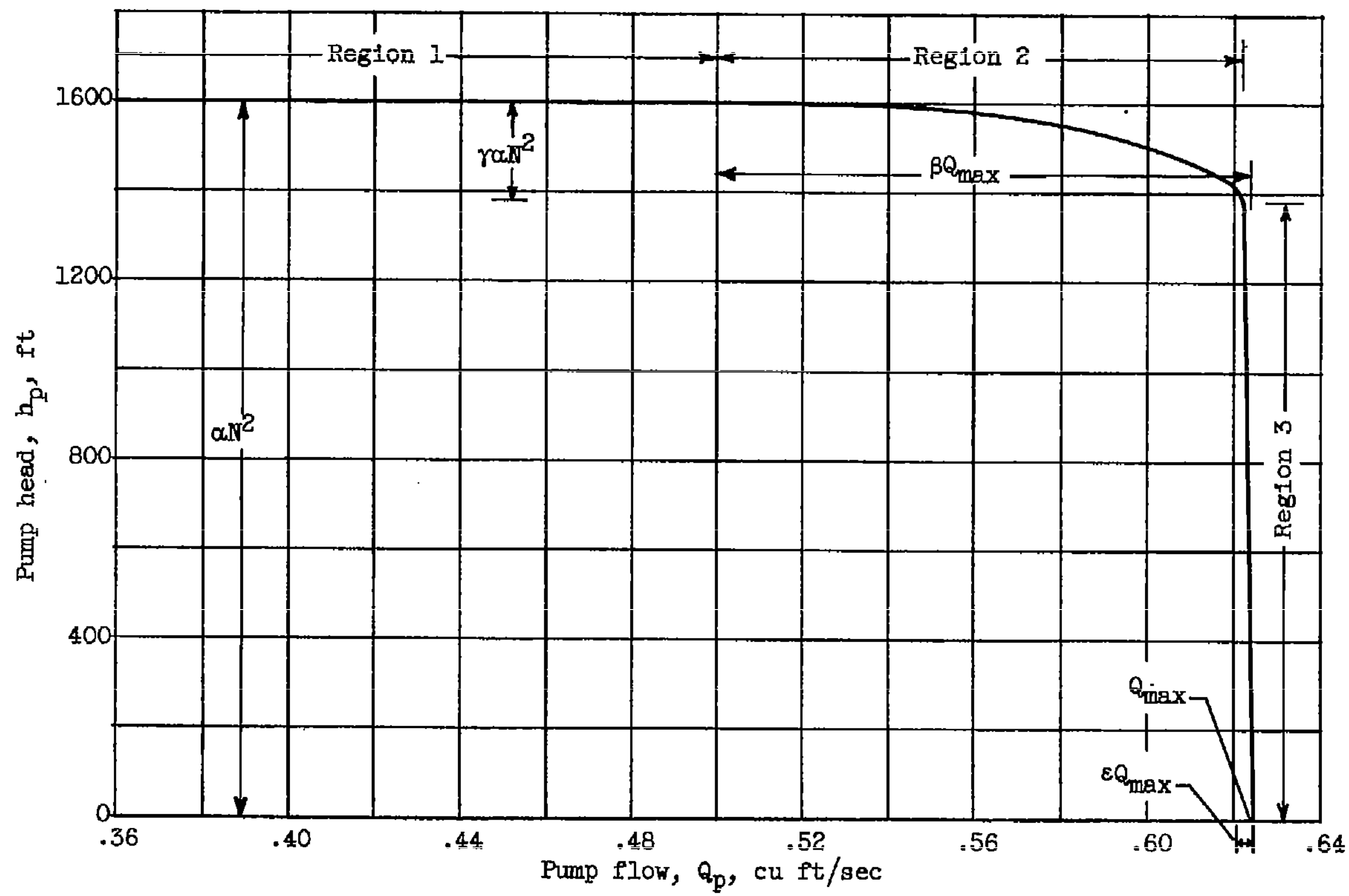


Figure 2. - Typical pump operating curve. Operating speed, 25,000 rpm.

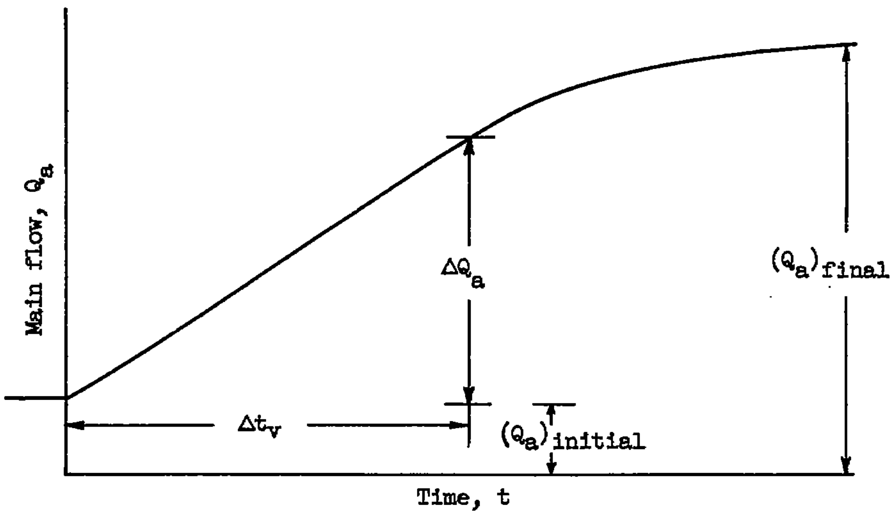


Figure 3. - Relation between volume flow and time for ideal flow.

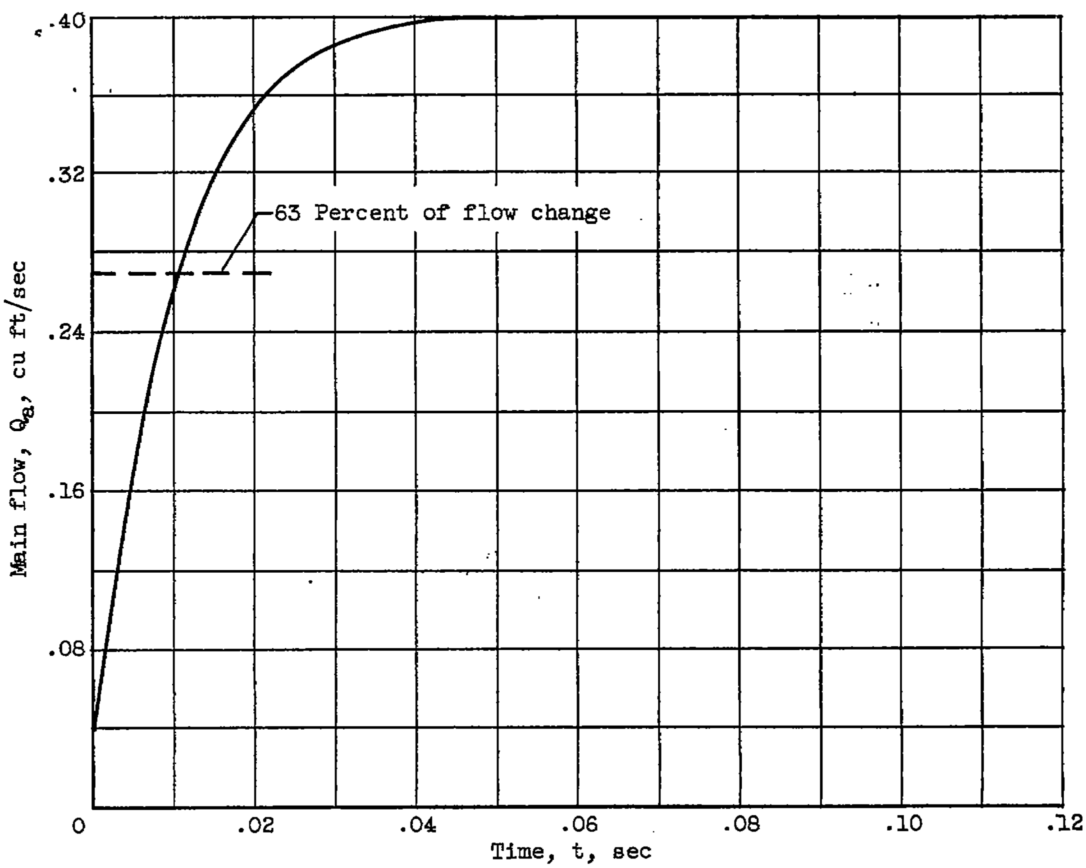


Figure 4. - Time history of main flow with step valve.

4516

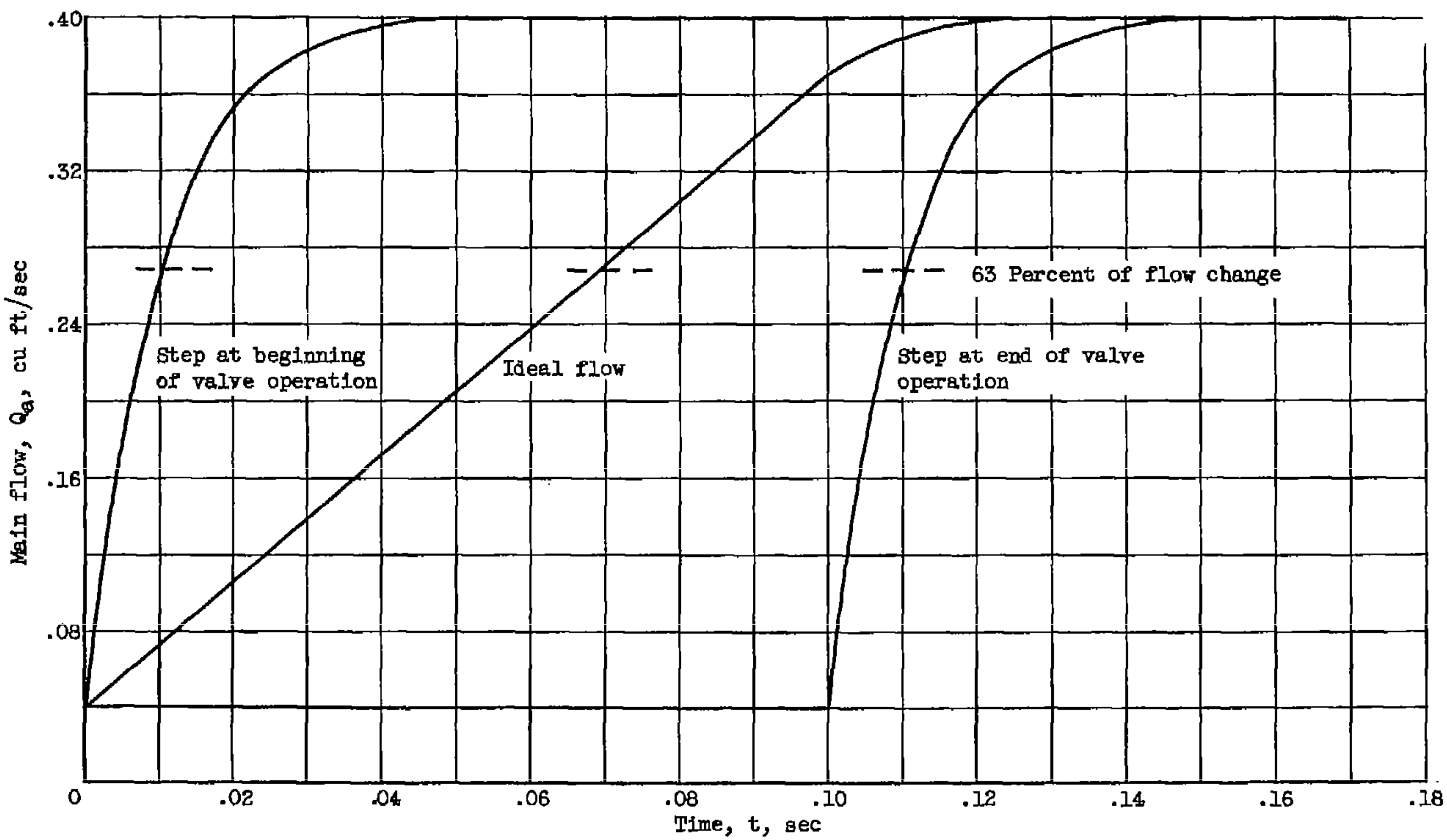


Figure 5. - Main flows for maximum and minimum suppression heads.

4516

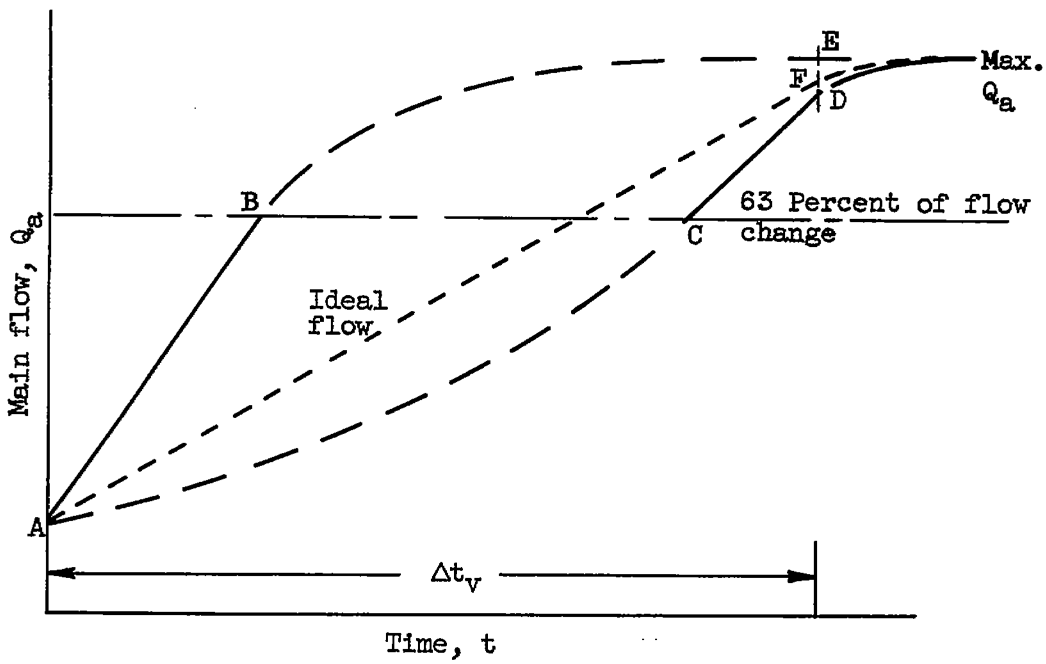


Figure 6. - Optimum flow variation for two values of flow response time. Ideal flow is included.

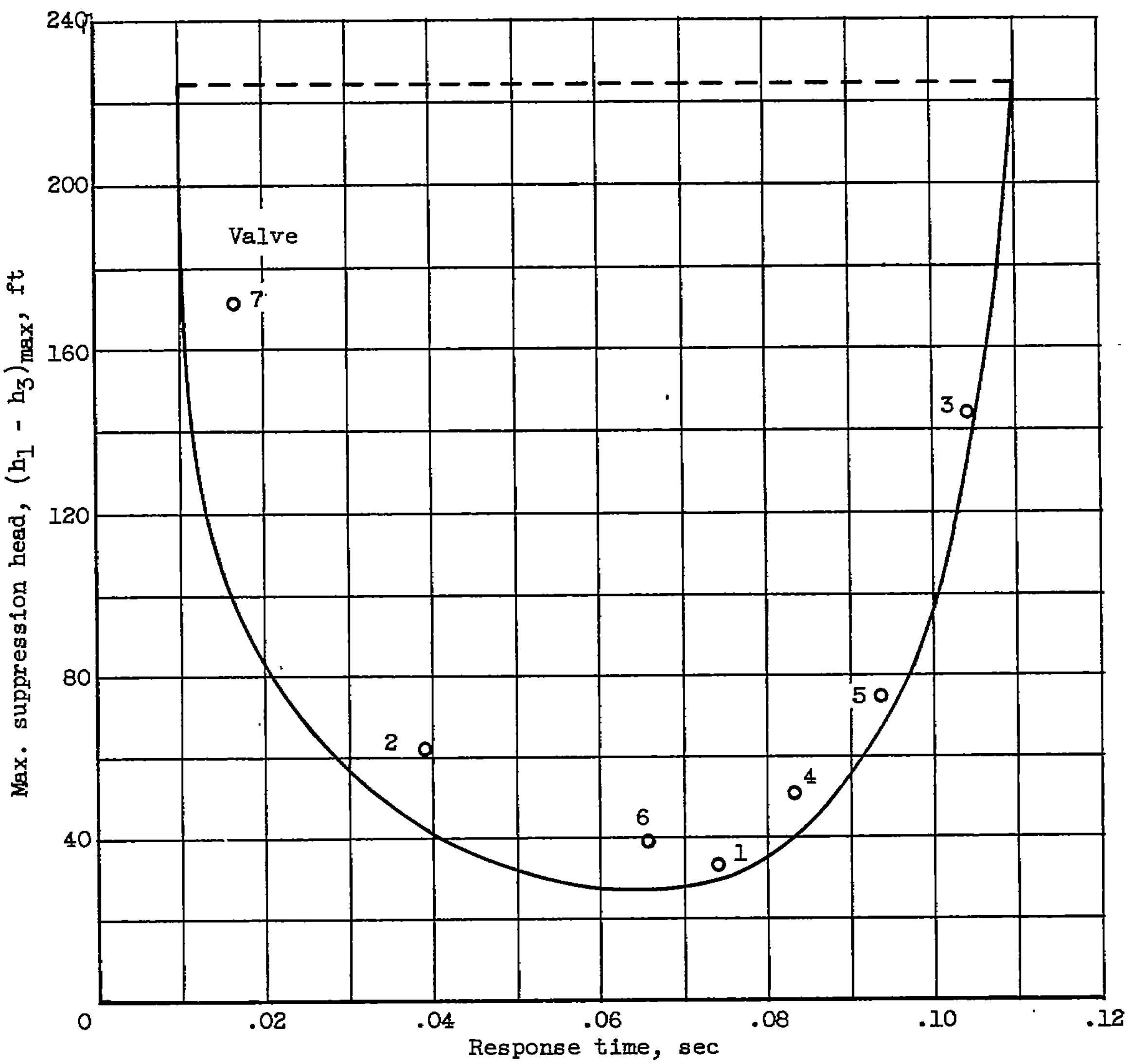


Figure 7. - Relation between maximum suppression head and main-flow response time.

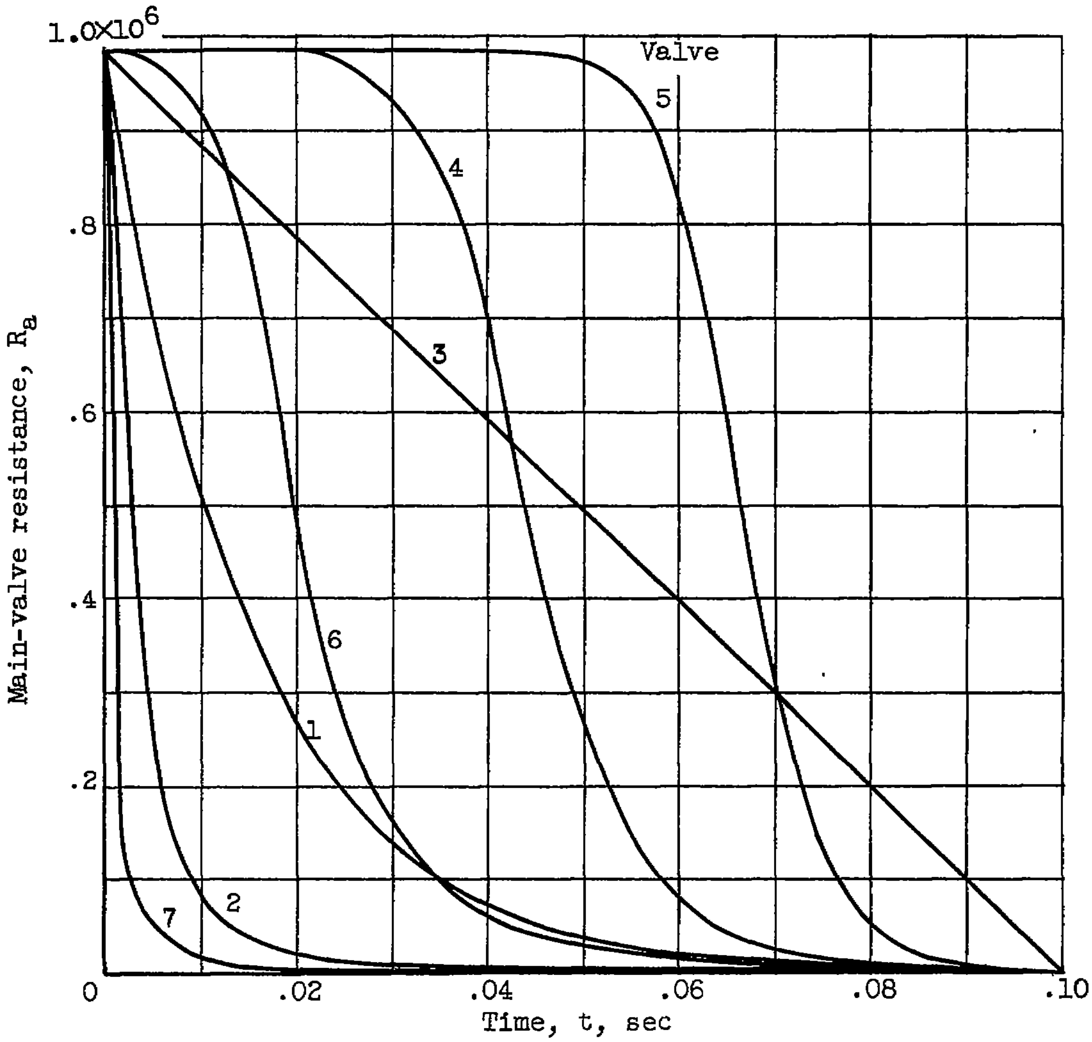


Figure 8. - Variation of main-valve resistance with time.

4516

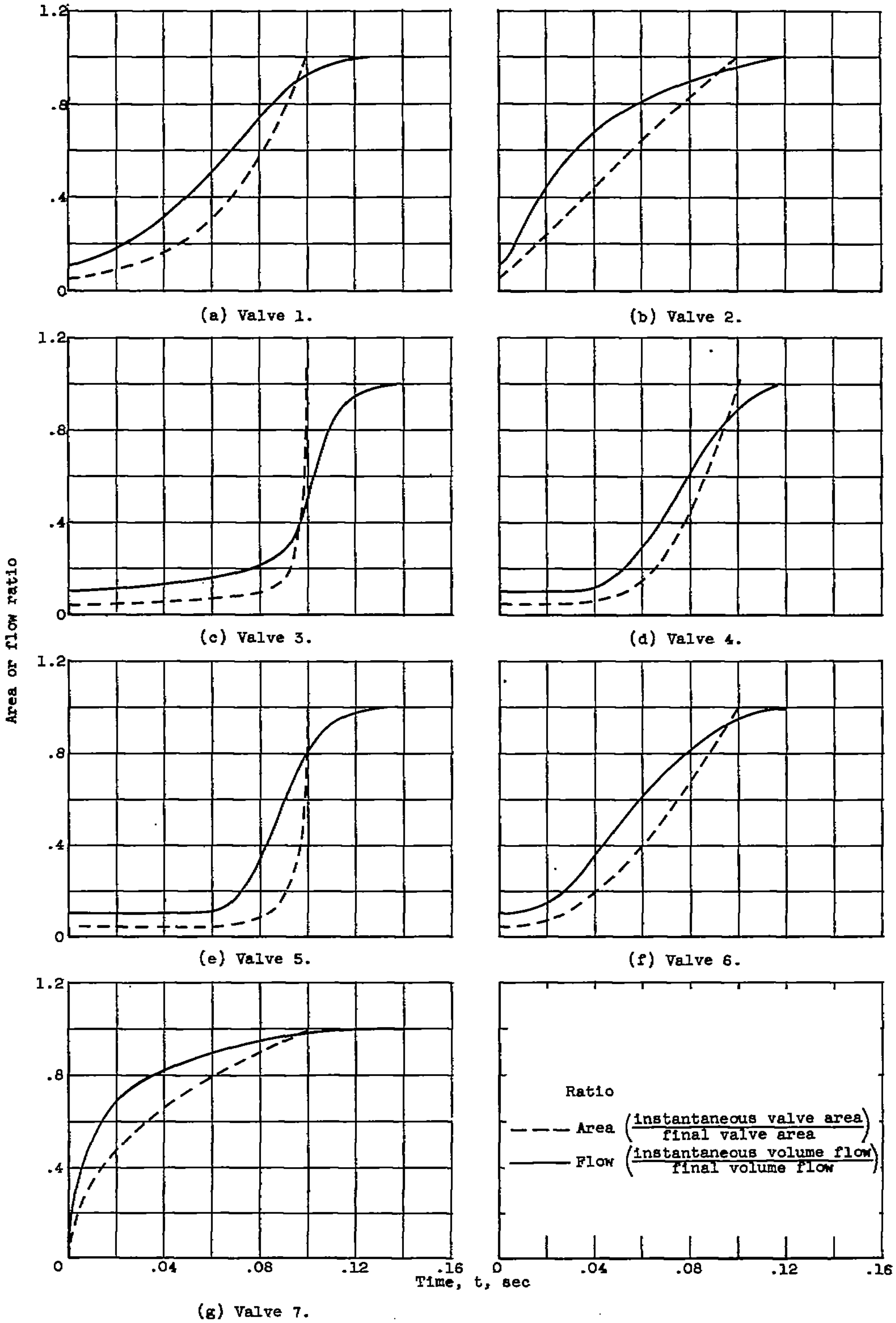


Figure 9. - Variation of main flow and approximate variation of main-valve area with time for valve resistances shown in figure 8.

4516

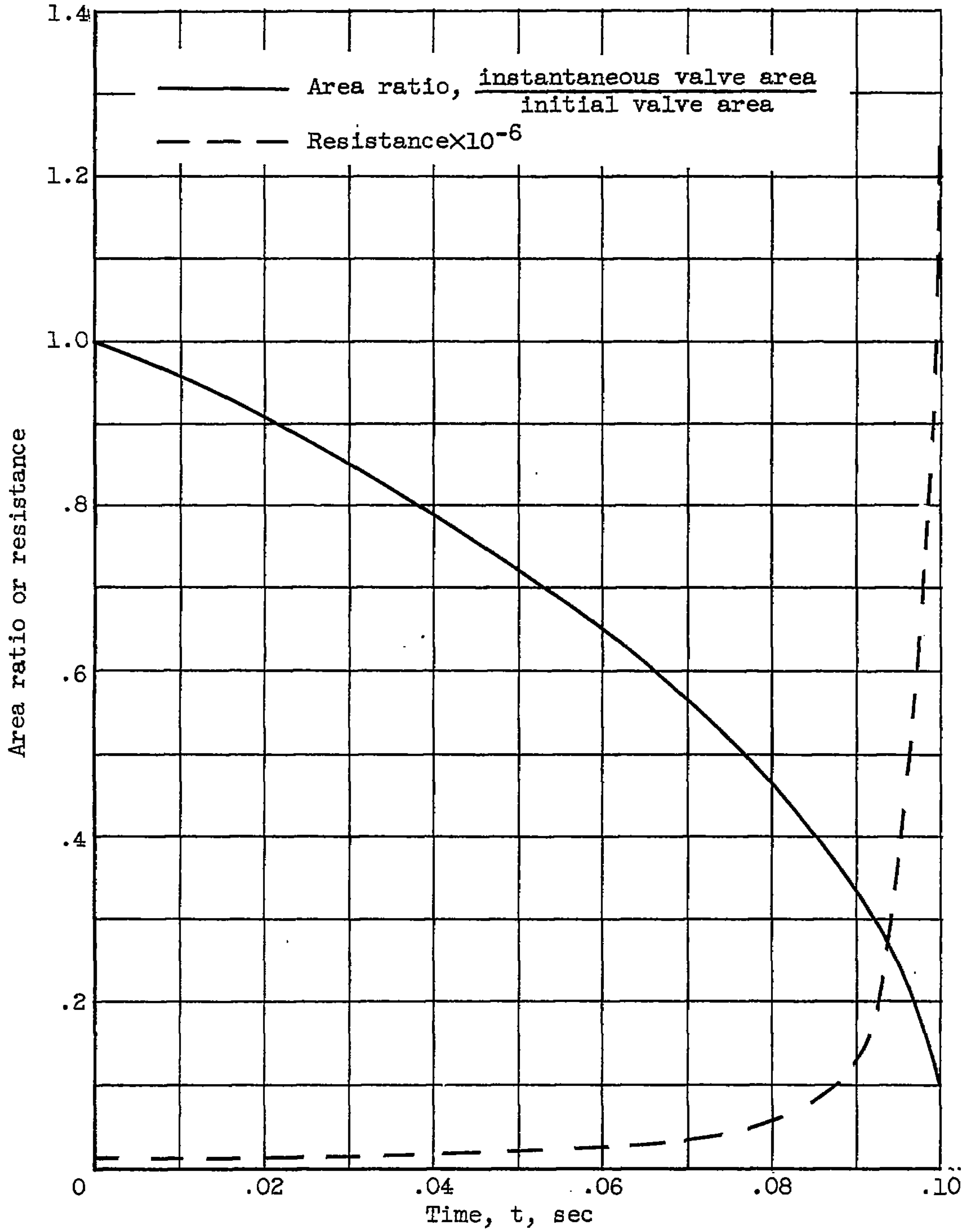


Figure 10. - Bypass-valve characteristics.

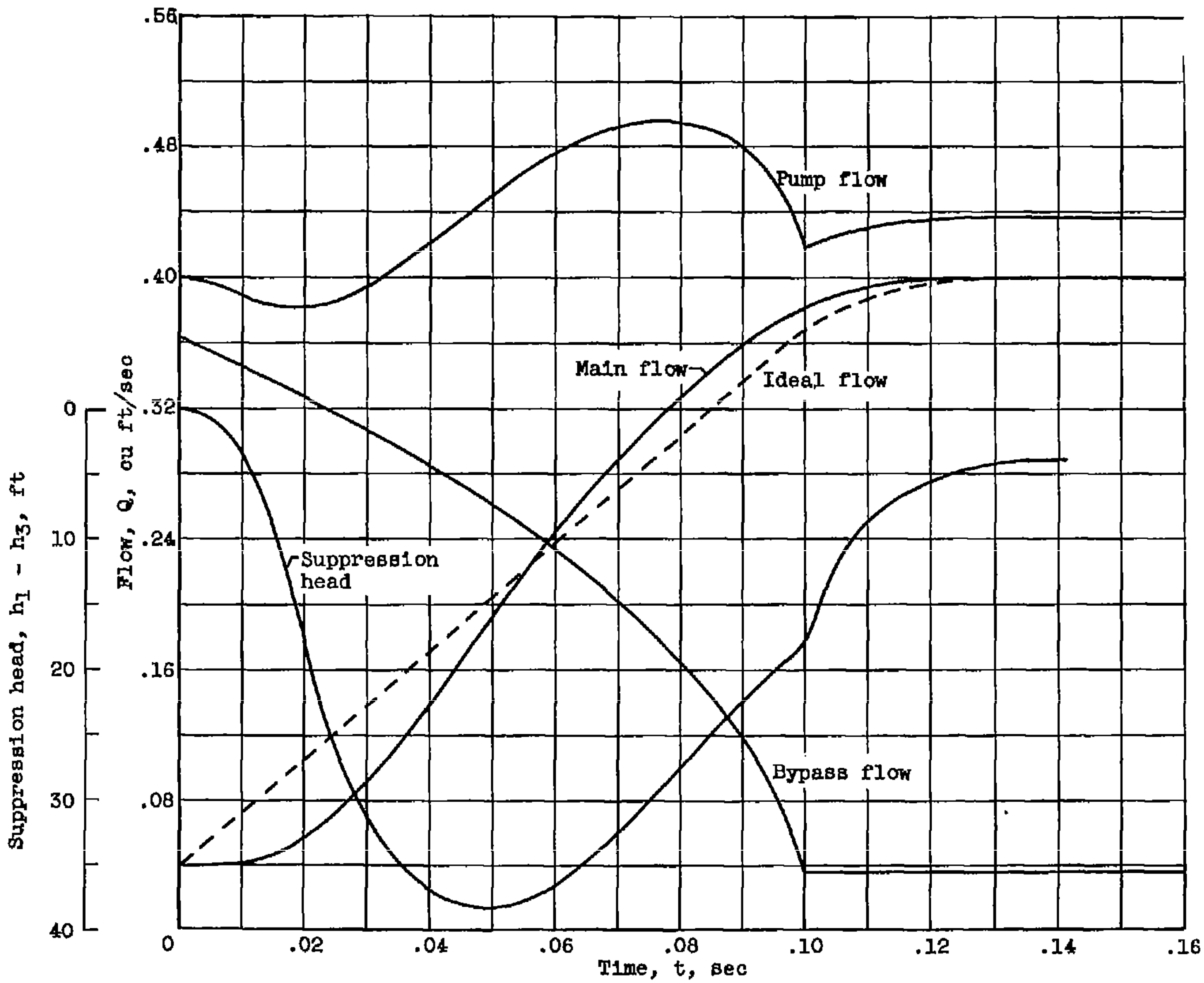


Figure 11. - Characteristics of low-suppression-head configuration.

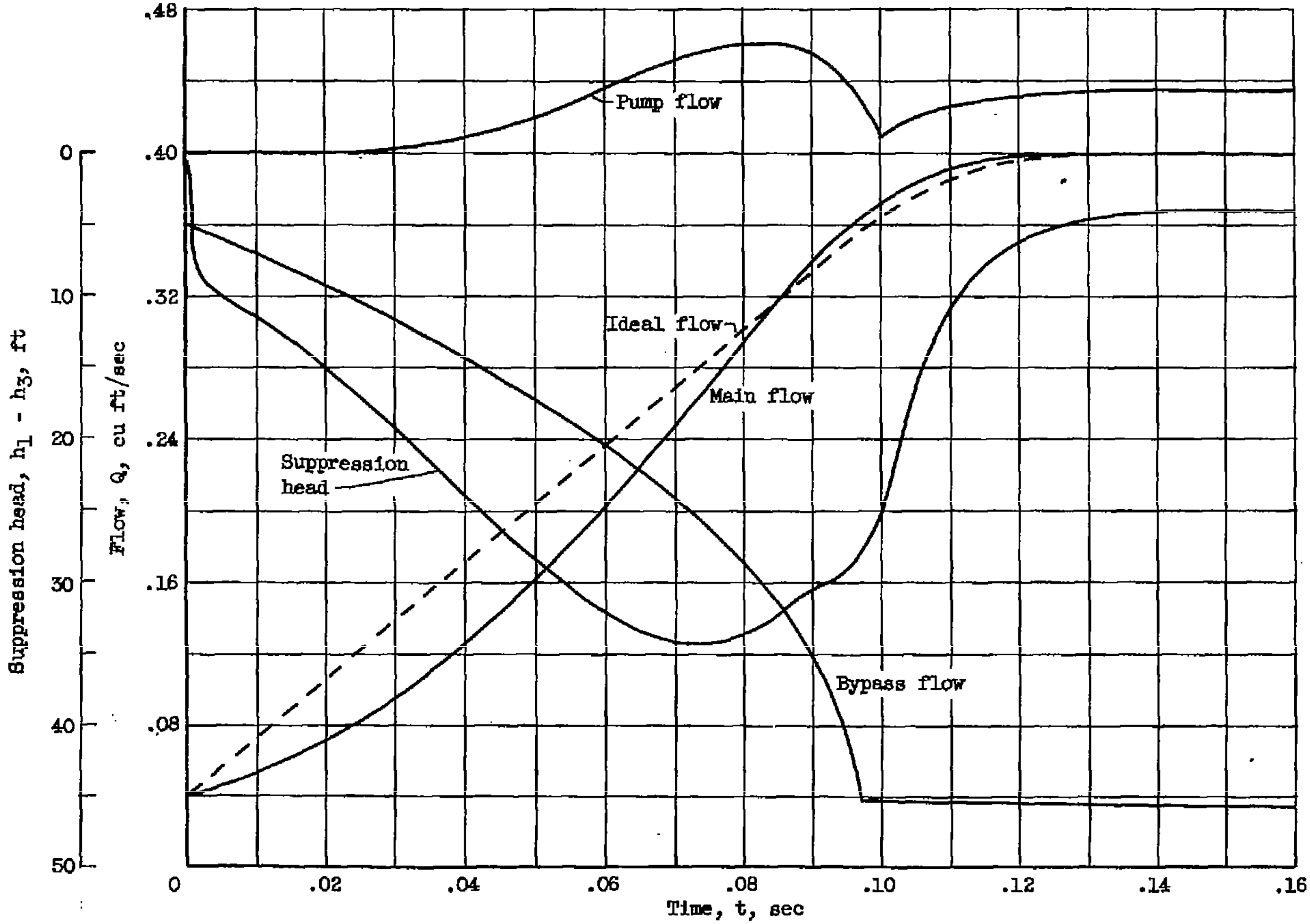


Figure 12. - Flow characteristics and suppression head for valve 1.

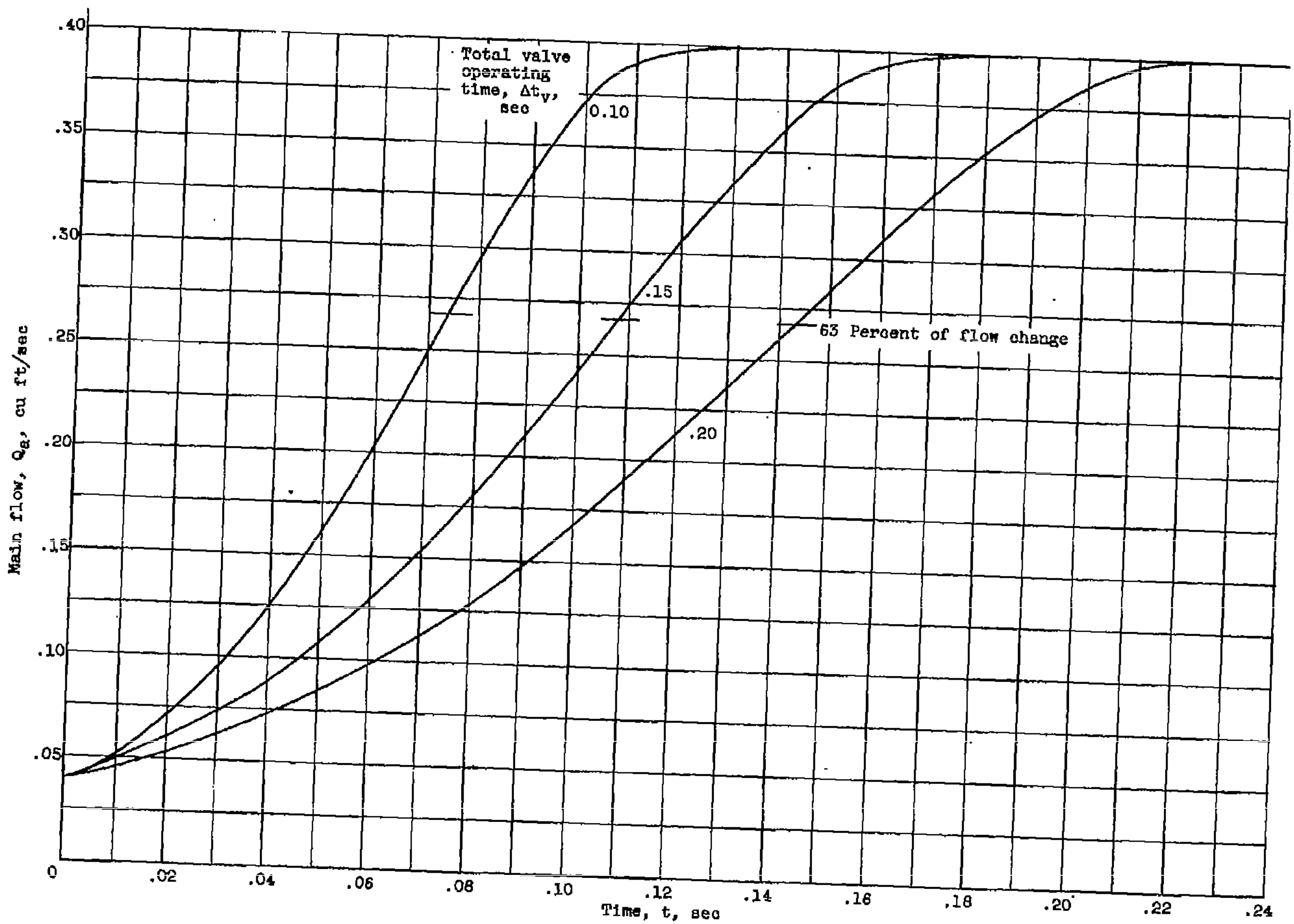


Figure 13. - Variation of main flow with time for three total valve operating times. Main valve 1.

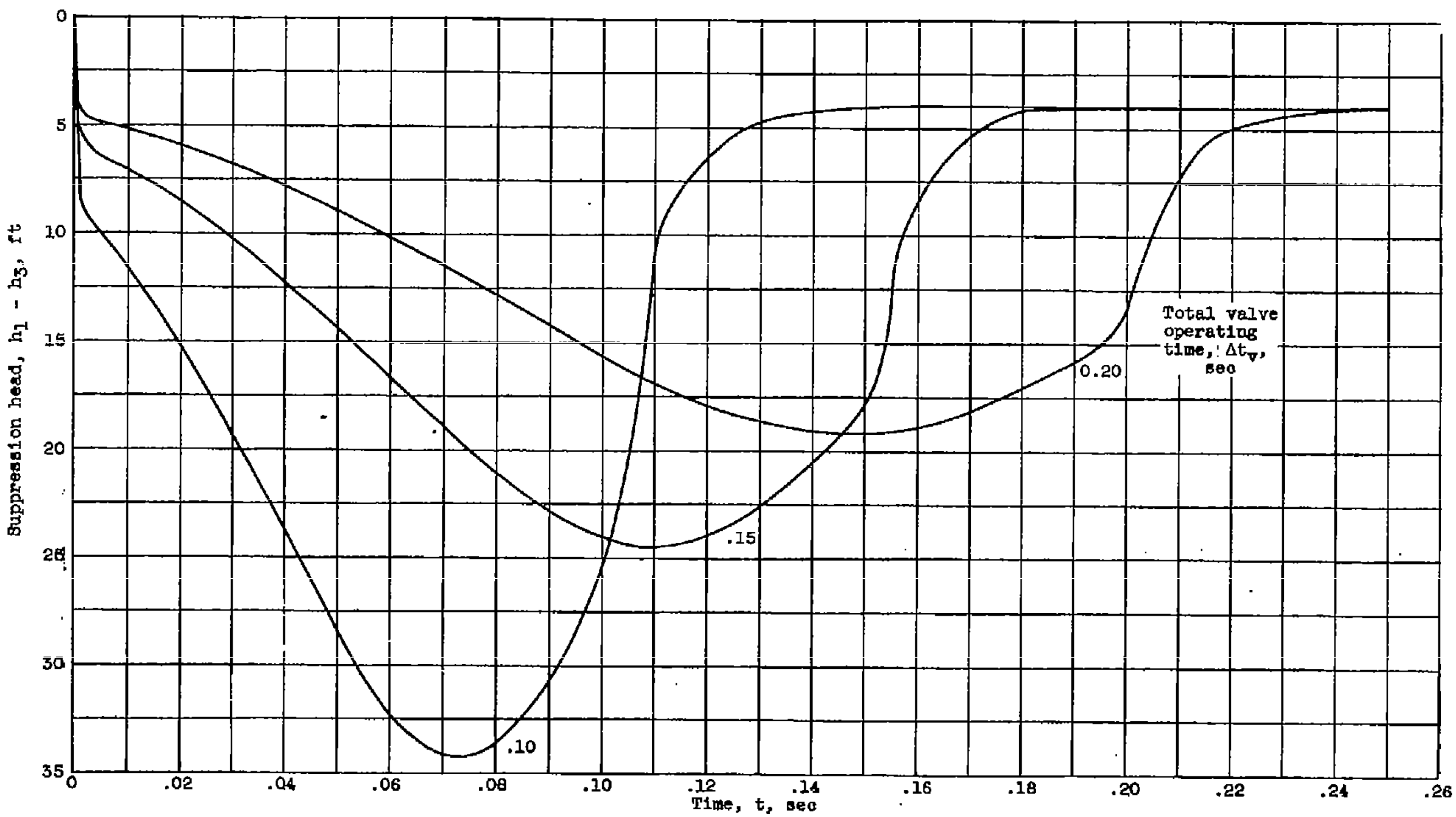


Figure 14. - Variation of suppression head with time for three total valve operating times. Main valve 1.

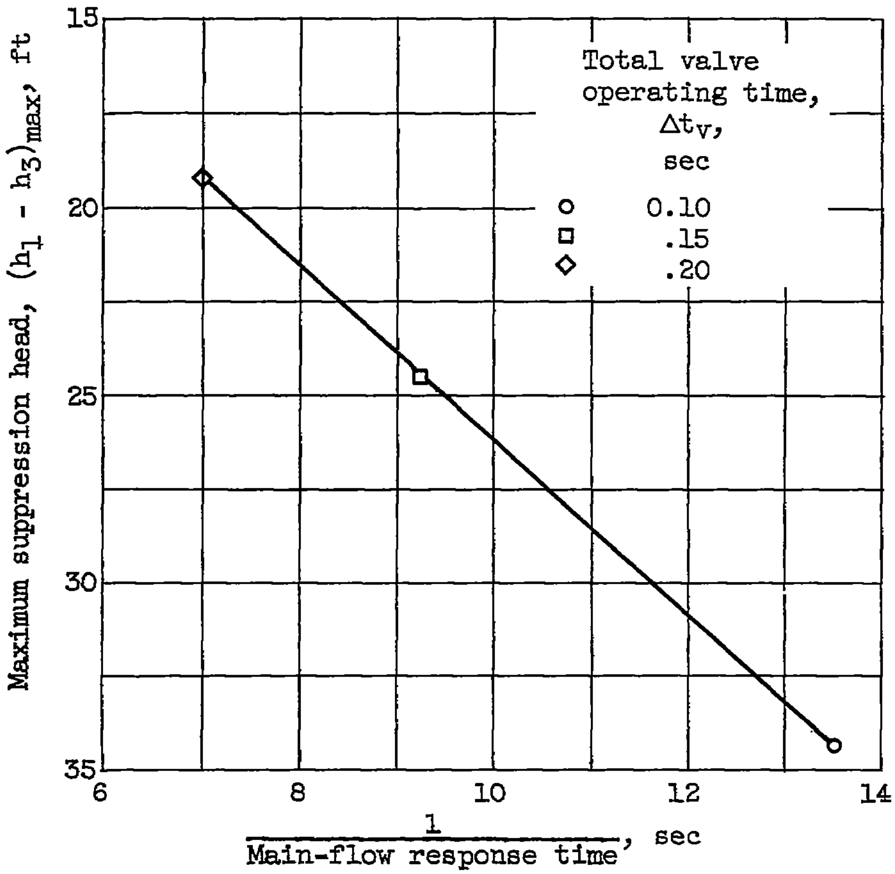
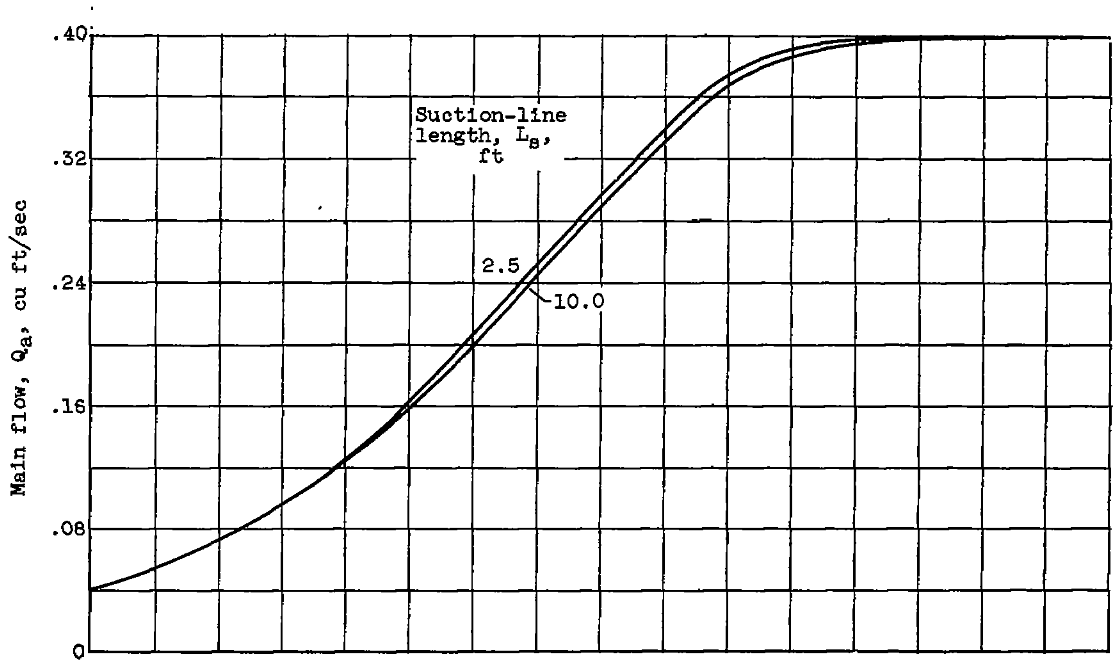
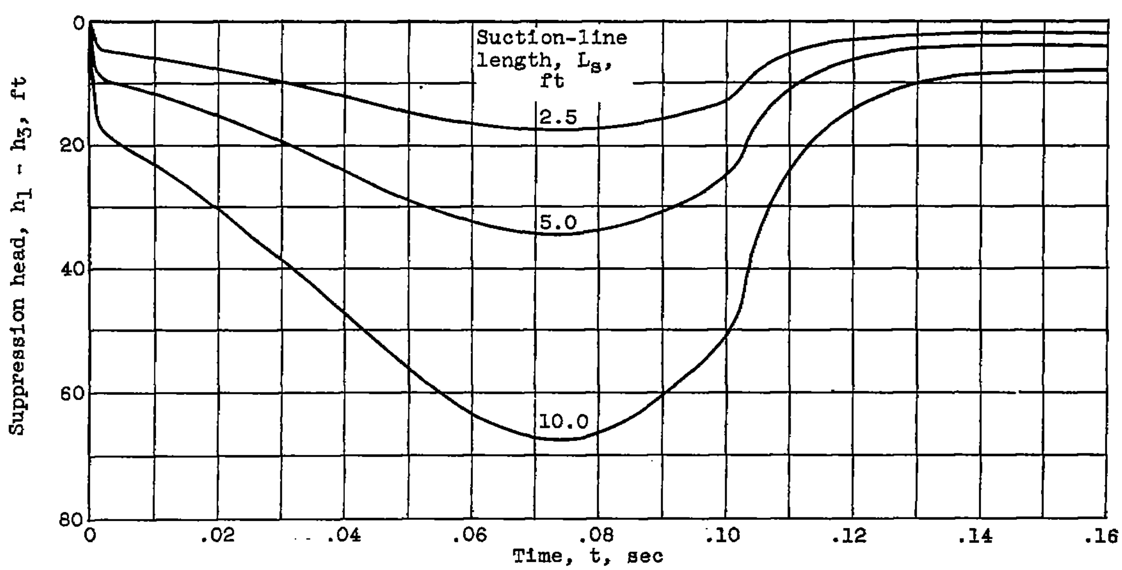


Figure 15. - Variation of maximum suppression head with inverse main-flow response time for three total valve operating times. Main valve 1.



(a) Main flow.



(b) Suppression head.

Figure 16. - Effect of suction-line length.

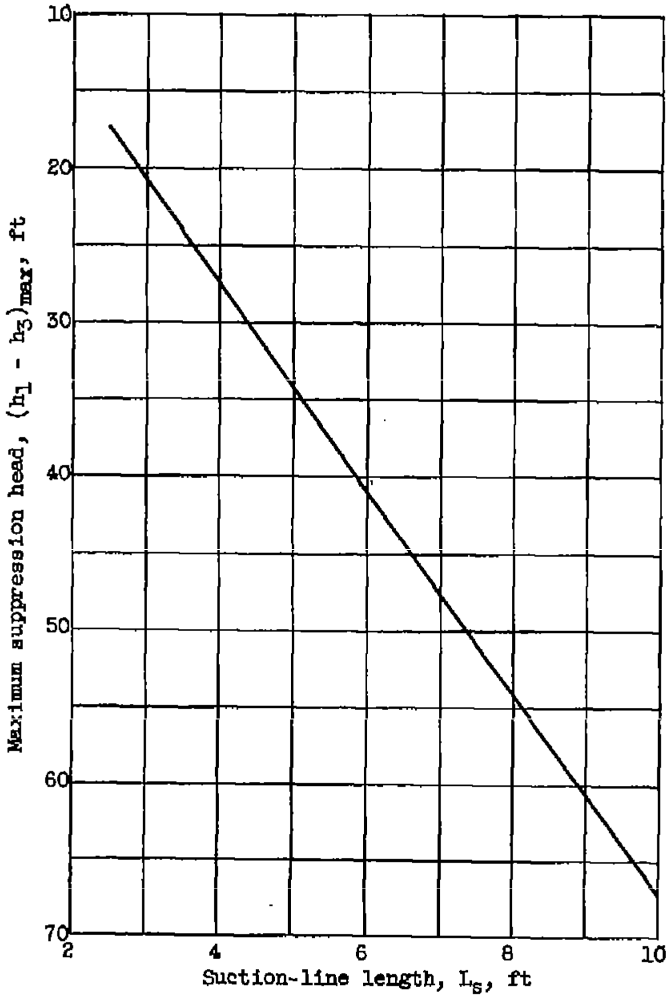


Figure 17. - Maximum suppression head as function of suction-line length.

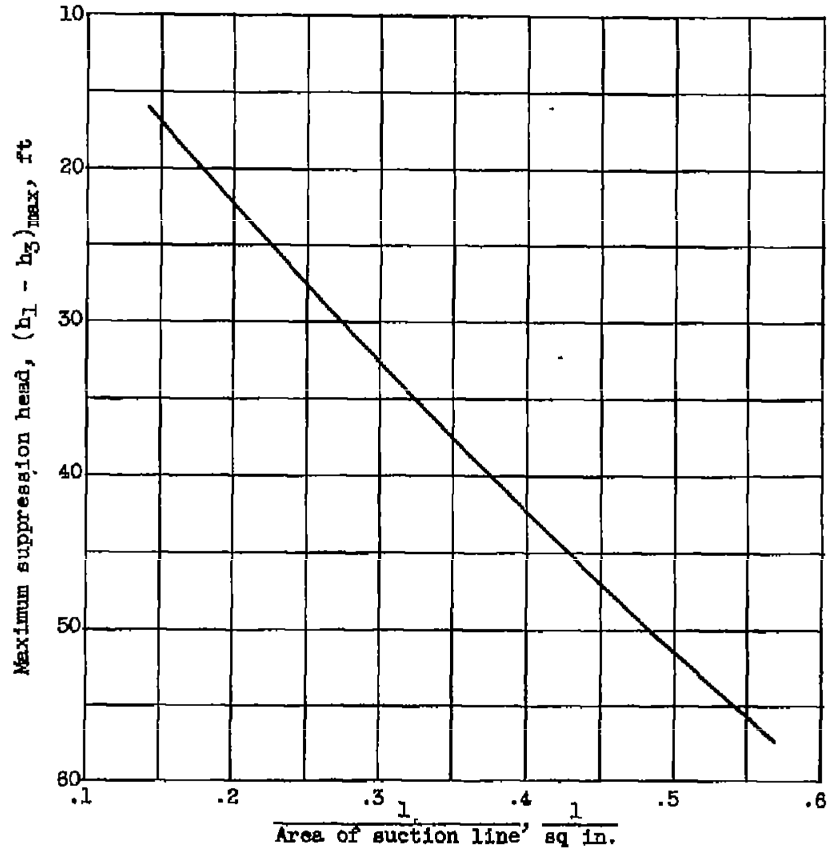


Figure 18. - Maximum suppression head as function of suction-line area.

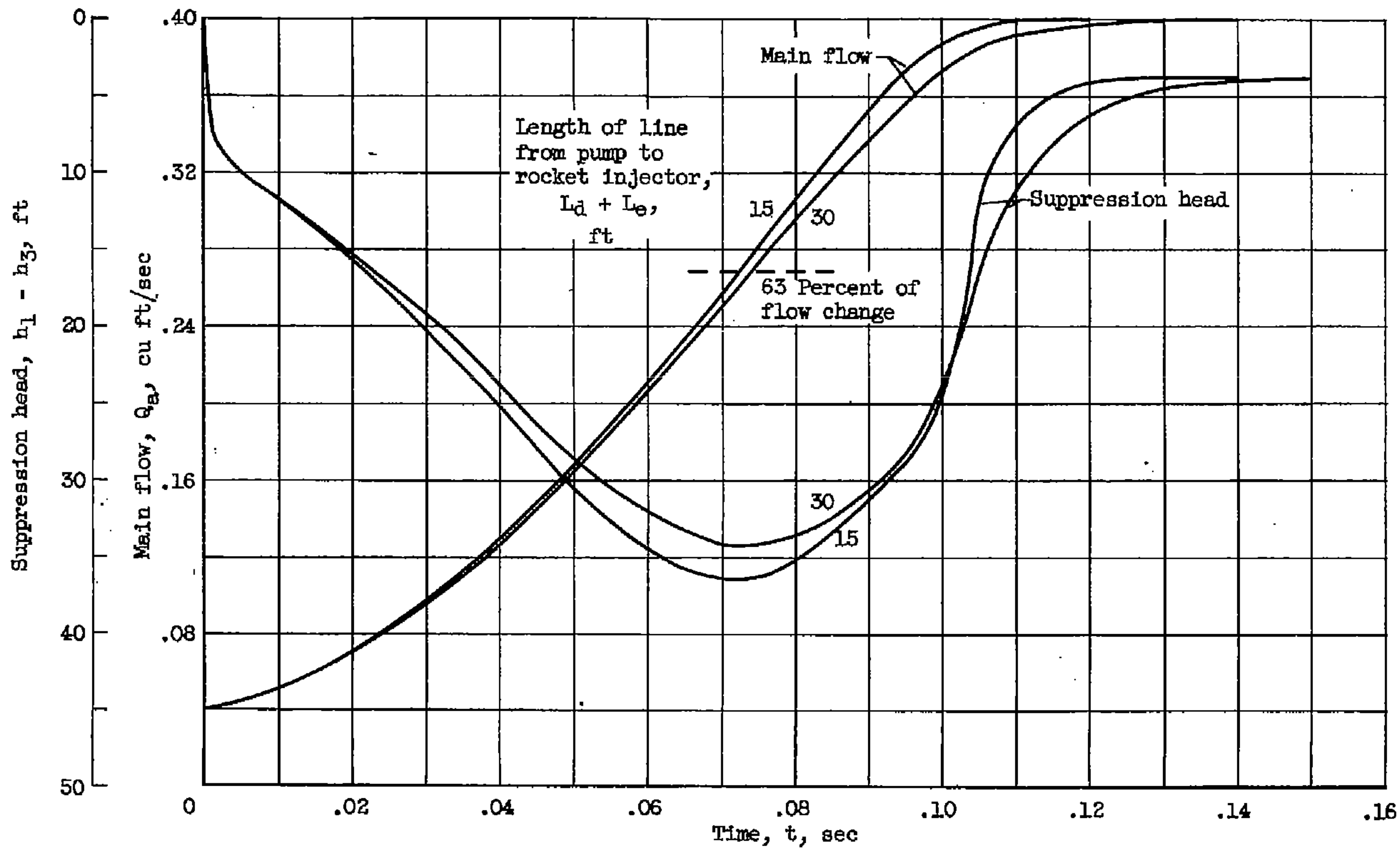


Figure 19. - Main flow and suppression head for two different lengths of line from pump to rocket chamber.